Upside down ∀s and algorithms – computational formalisms and theory

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1. Motivation

The time delay as Internet signals cross the Atlantic is about 70 milliseconds, about the same time it takes for a nerve impulse to run from your finger to your brain. Parallels between computation and cognition run as far back as computers themselves. Although at first it feels as if the cold, abstruse, more formal aspects of computation are divorced from the rich ecology of the human–computer interface, the two are intimately bound. Mathematics has also been part of this picture. Indeed the theory of computation pre-dates digital computers themselves as mathematicians pondered the limits of human reasoning and computation.

There are a number of aspects of this interplay between computation, mathematics and the human–computer interface.

First, understanding your raw material is essential in all design. Part of the material of human–computer interaction is the computer itself. Theoretical and formal aspects of computing can help us understand the practical and theoretical limits of computer systems, and thus design around these limits.

Second, diagrams, drawings and models are an integral part of the design process. Formal notations can help us sketch the details of interaction, not just the surface appearance, of an interactive system and thus analyse and understand its properties before it is built. This is the area that is typically called 'formal methods' within HCI, and we'll look at an example of this in section 2.

Third, various techniques from mathematics, simple counting to sophisticated equations, may be used to reason about specific problems in HCI. In this book we see chapters including Fitts' Law, a logarithmic regression; information foraging theory. which involves differential equations; not to mention the heavy reliance on statistical modelling and analysis of virtually all quantitative empirical work.

Finally, the design artefact of HCI involves people and computers working together within a socio-technical whole. Amongst the many political, social and emotional aspects of this interaction there is also an overall computational facet. The theory of computation has, from the beginning, spanned more than mere mechanical computation and conversely an understanding of digital computation can help us understand some of the complexity within rich organisational ecologies.

what is formal

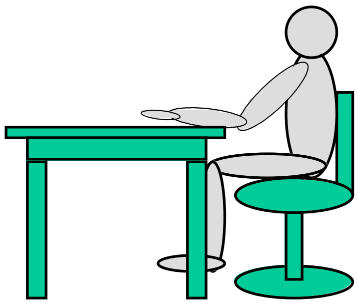
As with all words, 'formal' is used to mean different things by different people and in different disciplines. In day-to-day life formal may mean dinner jacket and bow tie or using proper language. That is, formal is about the outward form of things – a formal greeting may belie many emotions beneath the surface.

Taken strongly, formalism in mathematics and computing is about being able to represent things in such a way that the representation can be analysed and manipulated without regard to the meaning. This is because the representation is chosen to encapsulate faithfully the significant features of the meaning.

Let's see what this means in simple mathematics. One night you count the cockroaches on the wall – 213. The next night you count again – 279. Now because 279 is bigger than 213 you can assert that there were more cockroaches on the second night than the first. How do you know? Did you line up the cockroaches in queues? No! You know because you compared the numbers. The numbers 213 and 279 represented faithfully the significant feature of the cockroaches. Of course, the cockroaches on the first night might have been bigger, a different colour, more friendly, so the numbers don't capture everything.

Even looking at something as simple as cockroaches we can see both the power and limitation of formalism. It allows us to discuss certain features succinctly and with precision, but without the things they represent being present. However, by its nature it excludes features that are not represented

When we write, when we speak, when we draw, symbols stand for things that are not present. In interfaces a drawing of a potential screen design allows us to discuss that design even though it is not there. However, the drawing does not allow us to discuss the dynamics (perhaps a storyboard might for a single scenario), or the effects of actions on underlying data, on the world or on other users. Certain formal descriptions of a system allow this. Think again of the cockroaches. If we were comparing cockroaches in two different rooms, it would, in theory, be possible to collect them in jam jars and then line the cockroaches up to see which room had more. However, if we had collected the first night's cockroaches in a jam jar we would not have had 279 cockroaches the second night to collect as some would be stuck in the jam jar. The formalism of counting not only makes the comparison easier, it makes it possible.



Formal descriptions often include place holders for unspecified things, for example, a furniture designer may draw a homunculus with limbs in the right proportions. The homunculus stands for *anyone* of the relevant height, arm length etc. Notice how powerful this is, not only can we talk about a specific person who isn't present, but anyone at all with particular properties. This is used to particular effect in mathematics where we can say things like: "for any number n, n+1 is greater than n". Here the place holder 'n' stands for any number whatsoever. This power to talk about an infinite amount of things (every number!) in a finite expression, is one of the things that makes mathematics so powerful.

Because the real thing does not need to be present, it may be something abstract, including something like a sequence of interactions. A storyboard would allow us to talk about one example of a fixed sequence of interactions, whereas in a formal system we can usually talk about things we would like to be true of any interaction sequence. And more abstract again we can talk about any system of a particular kind and all the possible interactions a user could have with that system For example, Mancini and I worked on issues surrounding undo and were able to show that any system with particular kinds of undo would need to be implemented in effectively the same way [[DML97,DM97]].

when you count cockroaches you have to decide what counts as a cockroach

Finally formalisms force you to think. It would be easy to look at the wall one night and think "there are more cockroaches than yesterday". Perhaps the wall just looks more densely covered. However, when we start to count the cockroaches we need to be a lot more clear. What counts as a cockroach – should I include that mess where I hit a cockroach with my shoe? The cockroaches keep running in and out of cracks, do I mean every cockroach including those hidden in the cracks (perhaps I could put spots of paint on them), or do I just mean every cockroach that is out at a particular time, say 11pm?

Sometimes this forcing things into conceptual boxes is unproductive (think how many hours have been wasted arguing whether a platypus is a bird or a mammal), but often the very act of being precise makes us think more clearly about problems. Whilst using formal methods to model various aspects of interaction I have often found that it is the process of creating the model that is more important than anything I do with it.

the myth of informality

There is a fashionable tendency to decry the formal, mathematical or precise in favour of softer more informal techniques. This is always a popular stance, perhaps because of the rise of post-modernism, perhaps because people feel powerless, cogs in the unbending bureaucracies of modern society. The success of seminal books such as Suchman's 'Plans and Situated Actions' [[S87]] and Winograd and Flores' 'Understanding computers and cognition' [[WF86]] are, in part, because they appear to present atheoretic, highly contextual and aformal approaches. (In fact, both have strong theoretical stances of their own and were written in the context of an over-formal design environment at the time.)

As already noted there are many other disciplines and many aspects of HCI where this is the right approach, and attempts to formalise such areas are misguided. However, maintaining the opposite view, the myth of informality, and eschewing formalism completely is both illusory and ultimately dangerous.

The design process may well be informal, but the designed product is anything but. A computer is the most formal artefact possible, *all* it does is interpret meaningless symbols according to formal rules.

Maintaining the myth of informality means that the effects of the intrinsic formality of a computer system have not been considered during design. Would you buy a ship from an engineer who lived in the Sahara?

There are arguments for and against formalising certain phenomena, but not computer systems. Any design process that produces computer systems as part of its final product (albeit within a rich socio-technical whole) is dealing with formality.

The choice is simple: do we deal with the formality in knowledge or in ignorance?

chapter overview

The next section will examine some simple examples in order to demonstrate the utility of this approach. We will then look at some of the history of the use of formalism in mathematics and computing from Aristotle to Alan Turing. This will then lead to a more detailed review of the different ways in which formalism methods and computational theory have been used in HCI. In order to validate this a more substantial case study will show how adoption of a simple formal notation for dialogue design led to a 10 fold improvement in productivity and more robust systems. The final part of the chapter will establish the current state of the field and look forward to the future potential of these techniques, especially in emerging application areas such as ubiquitous computing.

2. First steps

In order to show some of the aspects of formal methods in interaction design, we'll look at the way simple diagrammatic formalisms can help us examine two simple devices, a wristwatch and an alarm panel. From these we will see some important general lessons about the advantages of using more formal techniques.

two examples

Some years ago my daughter had a digital wristwatch. The watch had two buttons and came with a set of closely written instructions covering a slip of paper about 2 inches wide (5cm) and 15 inches long (40cm). Figure 1 shows one diagram on the paper (much larger than life size!). The two buttons were labelled A and B and this diagram was intended to show the main functions of button A.

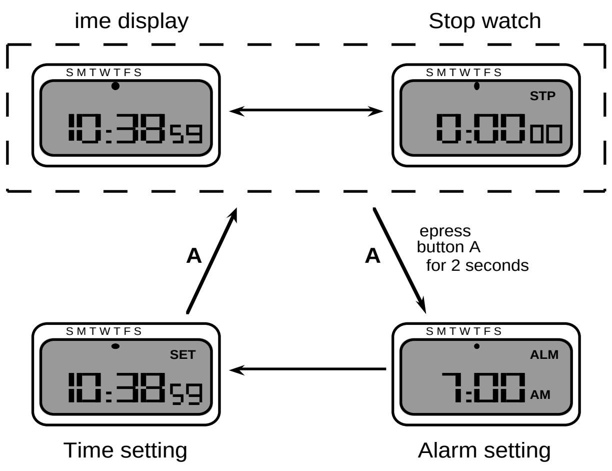


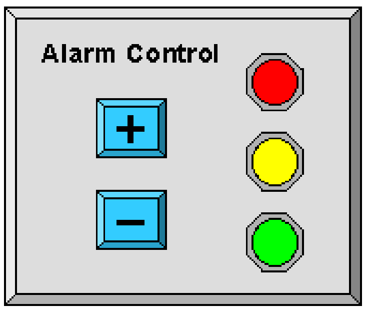
Figure 1. Digital watch instructions

From this diagram we can see that button A switches the watch between four main modes:

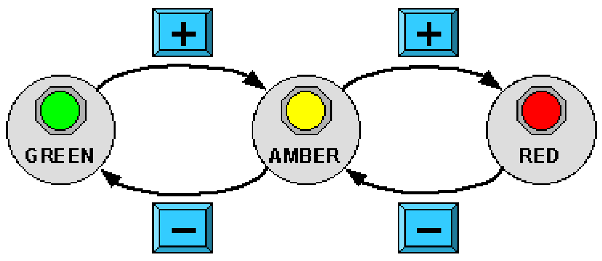
Time display:	normal mode showing current time and day
Stop watch:	timing to seconds and hundredths of a second
Alarm set:	to set the built in alarm
Time set:	to set/update the time

This diagram, produced as part of the user documentation, can be seen as a state transition network (or STN), a type of formal specification of part of the functions of the watch. STNs have been used extensively in interface specification. They are often written as small circles or boxes representing the potential states of the systems with arrows between the states showing the possible transitions. The arrows are labelled with the user action that initiated them. Describing states is always difficult and the digital watch instructions do this by simply drawing what the watch will look like in the state.

Consider an even simpler interface – perhaps the security control panel at a top secret establishment (figure 2.i). It has two buttons labelled '+' and '–' which control the current alarm state which can be at three levels: three lights: green, amber or red.



(i) control panel



(ii) state transition network

Figure 2 Top secret control panel

Imagine we have been given a state transition network for this panel (figure 2.ii).

Just looking at the STN – without knowing what it is about – we can ask some questions: what does the '+' button do when the state is red, what does the '–' button do when the state is green? This is a formal analysis – we don't need to know what the STN means, but we can see just from the pattern that something is missing. In many applications, such as a program control for a television, we would probably want the

'+' button to cycle back round from red to green, but in this application – an operator under stress perhaps – we would probably not want to risk accidentally changing from red alert to green. The formal analysis tells us *that* something extra is needed, the contextual understanding tells us *what* it is.

Looking again at the digital watch there is a similar story. At first it looks as though it is complete. Only one button is being considered and the diagram shows what it does in every state. Or does it? What happens from the Time setting state – does it go to Time display, Stop watch or perhaps whatever it was in before going into Alarm setting? In fact, after a quick bit of experimentation, it turned out that it always goes to the Time display state.

Also, look at the transition labelled "Depress button A for 2 seconds". This suggests two things:

- time is important
- we need to think about depressing and releasing the button as separate events.

This immediately prompts a series of questions:

- Does time matter in either of the 'setting' states?
- Do the transitions labelled simply 'A' mean 'when A is depressed' or 'when A is released'?

Further experiments showed that the transitions between 'Time display' and 'Stop watch' happened when the button was pressed. However, clearly the system 'remembered' that the button was still down in order to change to 'Alarm setting' state after 2 seconds. Figure 3 shows the full picture with the 'Time display' and 'Stop watch' states each split in two to represent this 'remembering'.

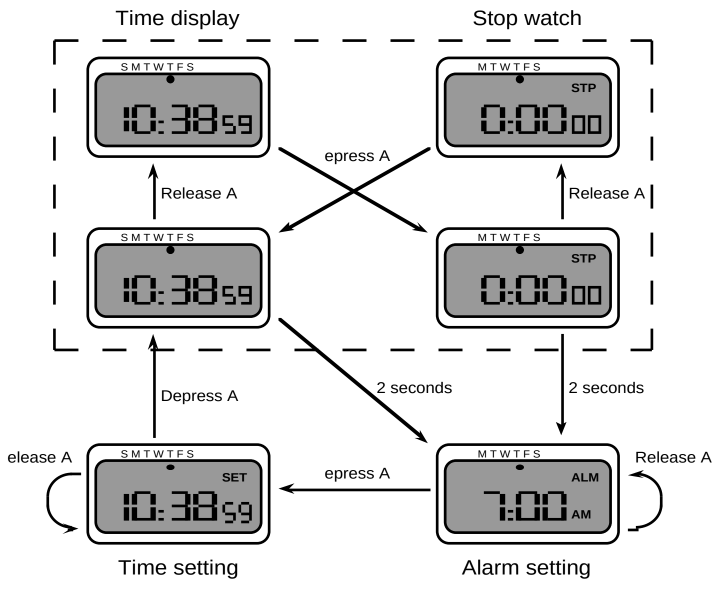


Figure 3. Digital watch states in detail

lessons

These examples demonstrate several key features of formal descriptions:

formal analysis – It is possible to ask questions about the system from the formal description alone. For example, in the alarm system, we noticed that '+' was missing from the red state. We didn't need to know anything about what the diagram meant (nuclear meltdown, or simply colour chooser), we just used the **form** of the diagram alone.

early analysis – In the case of the watch we actually had the artefact, but because we could look at the diagram we could have performed the same analysis before a prototype was available. Even with rapid prototyping this can save precious resources.

lack of bias – If we had been testing the alarm system without the formal analysis we might not have thought to try out 'unusual' actions like pressing '+' when the alarm was already in 'red' state. Testing is often biased towards 'normal' behaviour – what the designers expect the users to do. Formal analysis helps to break this bias.

alternative perspective – In a single diagram the formal description looks at the system with every different potential user input. Drawings of the watch early in design would show what it was like (but in greater detail) at individual moments of time. A fully working prototype would give a better feeling for what interaction with the watch was really like, but still does not let you see different possibilities except by repeatedly experimenting. So different representations allow us to see different things about a design. Furthermore different kinds of formal representation can allow yet more views on the artefact during design.

forcing design decisions – The watch example is taken from the user documentation and so would not be expected to show all possible fine behaviours (it would simply confuse). However, one wonders whether the detailed behaviour in figure 3 was the result of the designer thinking about what was wanted or arbitrary decisions taken whilst coding the internals of the watch. Using formal representations the designer is forced to make these user interface decisions explicitly and to communicate those decisions to the implementor.

On the other hand the example in figure 3 only shows the behaviour of one of the two buttons of a pretty simple device. Formal descriptions, by making you be precise, can become complex through sheer level of detail. However, this extra detail is not being created for the sake of the formal description but is simply exposing the extra decisions that will be there in the final artefact. If they aren't there in the design documents they will be in the code!

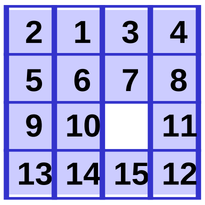
3. Scientific foundations

In this section we'll look at the history of formalism and how this feeds into current uses of formal notations, formal methods and general computational theory. Much of this story is about the gradual discovery of the fundamental limits of human knowledge in general and computation in particular.

a brief history of formalism

The roots of formalism go back a long way, certainly to Euclid's axiomatisation of geometry and to Aristotle with the formalisation of logical inference [[R08]]. It is Euclid who had the most significant impact on mathematical education. It is amazing to think that Euclid's *Elements* was still being used as a high school text book 2200 years after it was written! However, it is Aristotle who is the more 'modern' thinker. Despite coming through the Platonic school of perfect forms, he was able to look to the world as it is – empiricism. But he also based this on two pillars of self-consistent and sufficient basic truths in theology and logic (mathematics).

Of course many things happened over the next two thousand years: Arabic mathematics with the introduction of zero and the modern digits; the development of calculus (differentiation and integration), which enabled the formulation of modern physics, engineering, economics and much more. However, this is not the story of mathematics, but formalism, so we'll skip forward to the nineteenth century.



Galois is one of my heroes. He set 'group theory' on a formal footing. Group theory is in one sense a very abstract branch of mathematics, yet, like numbers, it is an abstraction of common phenomena, such as the number of ways you can put a computer back into its box (upside down, rotated 180 degrees, etc.). Group theory can tell us that this slider puzzle cannot be solved, and it also underlies modern quantum mechanics, but above all Galois set it in a formal framework that is the pattern for much of modern mathematics. That is why he is an important step in the story of formalism, but not why he is my hero. Galois' theory did all this and also was the key to solving a whole range of open problems about the roots of polynomials and what can be constructed with ruler and compasses, many of these unsolved since the height of Greek geometry 2000 years previously. I particularly like the fact that his theoretical framework largely proves what you can't do, foreshadowing some of the great results of the 20th century. And he did all this before, at the age of 20, he was killed in a duel. Beat that!

The nineteenth century saw Cantor's formulation of set theory which is at the heart of virtually all mathematics and also much of 'formal' methods in computing. For example, it is set theory that gives us the vocabulary to talk about the set of possible keys the user could press:

```
Input = { 'a', 'b', ..., 'z', '0', '1', ... }
```

From this and functions between sets we can talk about the set of states of a system, and the function that maps the current state to the new state depending on the current input:

```
update: State × Input → State
```

Although there were problems and setbacks, the nineteenth century in mathematics, as in so many fields, was one of rising optimism. At the turn of the century mathematicians' success was so great they began to ask fundamental questions of the discipline itself: could all mathematics be performed strictly formally, without insight or understanding, simply by playing out the rules? Various major steps were taken, problems in the theory of infinite sets were solved (problems uncovered by Bertrand Russell's paradox of the set of sets that don't contain themselves). Whitehead and Russell began a monumental work of proving basic mathematics with total rigour from logical foundations. This took a major volume to even get as far as statements such as 1+2=2+1, but progress was being made and the pair worked their way through several more tortuous volumes.

Not only was mathematics itself progressing, but throughout the 19th century mathematics was being used as the language of creation: formulating and integrating the basics of physics, especially in Maxwell's equations which brought magnetism, static electricity and electrical current within a single coherent mathematical theory. It seemed as though there would be no end to this progress except in that the complete and final formulation of all things was within human grasp.

Remember Aristotle's twin pillars: theology and logic. Towards the end of the nineteenth century Nietzsche, reflecting this spirit of humanistic optimism and self sufficiency, declared the 'death of God', and in the process discarded one of Aristotle's pillars.

But, by the 1920s the foundations of this optimism had been systematically undermined. The discovery of quantum mechanics and relativity showed that the complete comprehensive treatment of physics was not round the corner and that the world did not behave in ways that made sense at all. The First World War hardened an already growing post-centennial cynicism and laid to rest, with the corpses in Flanders, the belief in the moral ascent of man.

But Aristotle's second pillar stood firm and, in the cloistered halls of Cambridge, Whitehead and Russell laboured on.

Then in an obscure conference hall a young mathematician, Gödel, stood up and announced the death of mathematics. The last pillar had fallen.

the limits of knowledge

I said that Galois in proving the impossibility of many problems also prefigured the 20th century. The 19th century appeared to be an unstoppable chain of achievement leading to the belief that everything was possible. In contrast the successes of the 20th century have been in understanding what is not possible.

Gödel began this process in mathematics. He proved that any formal system was either incomplete (it couldn't prove some things even though they are true), or inconsistent (proves things that are false), and moreover, we can't tell which is true of a particular system, except by applying a different system ... and then we don't know about that one [[K67]].

Gödel showed that our knowledge in mathematics is patchy and full of holes, like a Swiss cheese. Not accidental holes because we haven't got round to finding something out yet, but fundamental holes because they can never be filled.

Given his fundamental reshaping of the intellectual landscape, Gödel should surely rank alongside Einstein. The physics of quantum mechanics and relativity can only be understood through their mathematics, but Gödel cast in doubt the very meaning of the mathematics itself. Gödel found his evangelist in Hofstadter, whose cult book "Gödel, Escher, Bach: an Eternal Golden Braid" popularised many complex issues of incompleteness, self-similarity and self-reference [[H79]].

the theory of computing

The roots of the theory of computation began at around the same time, much of it before the first electronic computers. Church and Turing worked on understanding what was possible to compute, whether by clerks in a bank or by a machine, but each dealt with very different models of computation. Church's lambda calculus is a sparse algebraic description of functions using only lines of λs, variables (x,y,...) and brackets combined with a small number of 'reduction rules'. In contrast, the Turing machine consists of a small device running over an infinitely long tape. The magic is that, in a milestone result, it was found that these two, although different in methods of operation, were exactly equivalent in what they could do. Other models of computation were also found to be equivalent. This is taken as justification (not proof) of the Church–Turing thesis (actually coined by Kleene) and variants, which is a broader hypothesis that all computation is fundamentally the same [[SEP02]].

he Halting Problem

Some computer programs finish their job quickly and others may take a long time, but do eventually 'halt' and give their result. However some programs simply never finish at all, they never 'halt'. Imagine you have bought a clever program (let's call it `will_halt`) from magisoft.co.uk that tells you whether other programs will eventually halt or not. You then write the following:

```
my_program:
    if will_halt(my_program)
                loop forever
    otherwise  stop
```

Does your program halt? If it does and `will_halt` works this out then your program will loop for ever, if it doesn't halt and will_halt works this out then it will stop. That is if it halts it doesn't and if it doesn't it does. Something is wrong! You sensibly return `will_halt` to magisoft as it clearly can't possibly work.

THowever, this early theory did more than that, but also found its own limits. In a variety of results, most famously the halting problem (see box), the theorists of computation found that there were certain very useful things that it was impossible to compute.

So, from the very beginning computation theory has considered its limitations.

complexity

As important as whether something finishes is how long it takes. If something is going to take 100 years to happen I don't think I'll wait around. A branch of computing called complexity theory deals with these questions "how long", "how much memory", "how many network messages" and in parallel computers "how many processors".

Let's think of an electronic newsletter. Each day I email it to my subscribers. If there are 10 subscribers there are 70 messages per week, if there are 100 subscribers there are 700 messages per week, for 1000 subscribers 7000 messages. the number of messages rises linearly with the number of subscribers. If there are n subscribers this is usually written 'O(n)' (the 'O' stands for 'order of').

Now let's imagine a mailing list. Each morning everyone reads all the messages on the list and sends one of their own to *everyone* on the list (including themselves). If there are 10 people then there will be 700 messages a week, 10 people each day sending 10 messages. If there are 100 people there will be 70000 messages (7 days x 100 senders x 100 recipients), and 1000 people leads to 7 million messages per week. This time the number of messages rises with the square of the number of people. This is written '$O(n^2)$'.

Some problems are even worse. Imagine you are a manager trying to work out who should do which job. You decide to try every possible combination and each day assign people accordingly. Say there are 2 people, Ann and Bob, and 2 jobs, artist and builder. On day 1 you try Ann as artist and Bob as builder, on day 2 you try Ann as builder and Bob as artist. Two days and you've tried the lot. Now imagine there are three people: Ann, Bob and Chris and three jobs: artist, builder and clerk. This time it is a little more complicated so you write out the schedule:

	Ann	Bob	Chris
day 1	artist	builder	clerk
day 2	artist	clerk	builder
day 3	builder	artist	clerk
day 4	builder	clerk	artist
day 5	clerk	artist	builder
day 6	clerk	builder	artist

Notice this time 6 days for 3 people. When Dave joins the firm you branch out into medicine and take 24 days to try 4 people in 4 jobs. You grow further and find that 5 people in 5 jobs takes 120 days, and 6 people in 6 jobs takes 720 days. This number is growing as the factorial of the number O(n!).

At this point you enlist Pete (who you know is a good programmer!) to write a program which, based on data in the personnel database, simply simulates assigning people to roles and works out the best combination. Pete is good. The program only takes a microsecond to simulate each assignment, so you test the software by running it against your existing 6 people and in 720 microseconds, less than a millisecond, it gives the answer and agrees with the real experiment. Now that you have solved your personnel management problems you happily grow further. When you have 10 employees and 10 jobs the system checks and finds the best combination: in less than 4 seconds (3.6288 seconds to be exact) it gives the answer. As you grow to 11 and 12 employees it seems to be a little sluggish responding, but you put this down to the monthly accounts running at the same time. However, when you hire your 13th employee things start to go wrong. You wait and wait, then ring Pete and get his answering machine; just as he rings back, an hour and three quarters later, the computer comes back to life and gives the answer. "Ah well, it's just one of those funny things computers do" you agree. On the 14th employee you set it going in the morning and then go out to client meetings for the rest of the day. When you get in the next morning the computer screen looks frozen, perhaps it has crashed, you go to get a coffee and it is waiting there with the answer when you get back ... curious. But with the 15th employee things are clearly not right – the program seems to run and run, but never stop. Pete is on holiday for a fortnight and so you have to wait for his return. When he does you give him a ring and he says "ah, I'll have to think about it". The next day he calls and says, "look at the screen now" – and the answer is there. "That's amazing" you say "how did you do it". Then he admits that 15 employees simply took 15 days to complete, but that the 16th would require 242 days and if you want to grow to 20 employees you will need to wait seventy-seven thousand years for the answer and 24 employees would take 20 billion years, longer than the age of the universe. Ah well, they do say small is beautiful.

It is so easy to assume that computers are fast and can do anything, but if the underlying problem or algorithm doesn't scale no amount of speed will be enough. Problems like the last one that involve trying lots of combinations are known as non-polynomial as there is no polynomial (power of n) that bounds them. Even quadratic, O(n^2), algorithms grow in difficulty pretty rapidly (as in the mailing list) and to be really scalable one tries to find things that aren't too much worse than linear. For example, simple sorting algorithms are O(n^2), but there are faster algorithms that are O(n $\log_2$n). This means that sorting 8 items takes around 2x1 steps, 4 items takes 4x2 steps, 8 items 8x3, 256 items 256x8 and so on. This is longer than linear, but massively faster than a slow quadratic sort (32 times faster for 256 items and 100 times faster for 1024 items).

factorial

The factorial of a number is the product of all the numbers less than or equal to it, e.g. 6! = 6x5x4x3x2x1. It grows very rapidly, even faster than an exponential.

n	n!	
1	1	
2	2	
3	6	
4	24	
5	120	
6	720	
7	5040	
8	40320	
9	362880	
10	3628800	
11	39.9	million
12	479.0	million
13	6227.0	million
14	87178.3	million
15	1307674.3	million

good enough

Notice that the halting problem is about a program that can *always* tell us whether other programs halt; the personnel allocation program tries *every* combination in order to find the *best* solution. This is traditional algorithmics, deterministic methods that, excepting for simulations, dominated the first 25 years of programming and still form the main part of virtually all current systems and computing syllabi.

However, since the mid-1970s a second strand has emerged that is focused on *usually* getting an answer, or trying *enough* things to get a *reasonably good* answer. Methods in this strand include simulated annealing, neural networks and genetic algorithms, but there are also numerous more specialised algorithms that employ elements of approximation or randomness to solve problems that would otherwise be impossible or intractable. Many of these techniques are based on analogies with physical or biological systems: simulated annealing – the cooling of metals, neural networks – brains, genetic algorithms – natural selection, and recent work has been inspired by the behaviour of colonies such as ants or bees.

agents and interaction

The early theory of computing was also very much concerned with all or nothing computation – you give the computer a problem and then wait for the result. In HCI we are more interested in interaction where we constantly give little bits of input to a device and get little bits of response. To be fair, computational models that involve interacting agents have been around since the early days of computing. For example, cellular automata, made popular by Conway's 'Game of Life' [[[G70,BCG82]] were originally proposed by von Neumann, one of the pioneers of computing [[vN56,vN66]]. However, for a long time such interactions were mainly concerned with what happened *within* a computing system (typically as a computational model or parallel or distributed computing). This has gradually changed and it is now common to consider models of multiple agents interacting with one another, the environment and human users.

This view of computation as involving interacting entities has both practical and theoretical implications. My own studies of human processes emphasise the importance of initiative – who makes things happen and when [[D98,DRW02]]. This is certainly a key element in many areas of practical computing too, but until recently not a major part of the vocabulary of fundamental computation theory. Wegner has suggested that the fundamentally different nature of interaction may mean that interacting computers are fundamentally more powerful than all or nothing computation [[W97]].

Conway's Game of Life

Imagine a large board of squares such as a chess or Go board. Each square either contains a token (is alive) or doesn't (dead). Now for each square you look at its 8 neighbours and apply the following simple rules:

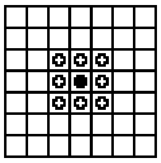
overcrowding: if more than three are alive the square becomes (or stays) dead

loneliness: if less than 2 are alive the square becomes (or stays) dead

birth: if exactly three are alive and the square is dead it becomes alive

otherwise the square stays as it was

The rules are regarded as operating at the same moment over the whole board (that is not in sequence).



If you regard each square as a small computational device, Life is a form of cellular automata. CA differ in how many states each device has (just two here, alive or dead) how many neighbours are considered, the geometry of the 'board' (2D, 3D, different shaped grids) and the complexity of the rules.

Working at a time when there were no physical computers, Turing was radical in ensuring that the models of computation he used were physically realisable. However, as real computers became available, this view was somehow lost and computation theory became more and more abstract. And to some extent we now see signs of a more grounded theory as computational devices are scattered amongst our everyday lives. I call this more physical view 'embodied computation' and find that it radically changes the way I personally look at computation [[D02c]].

notations and specification

Although a large part of mathematics is about manipulating formulae, it is remarkably laid back about notations. There are some fairly standard symbols (e.g. basic arithmetic, upside down ∀ for 'forall' and back to front ∃ for 'exists'), but these are regarded as a convention rather than a standard. In individual parts of mathematics and individual papers and books variants are used for the same thing and special notation introduced on an ad hoc basis.

Computing has always been more conservative about notation. Perhaps this is because of its connections with the more 'formal' parts of mathematics, perhaps because of links with electrical engineering where standardisation is more important (if a mathematician gets confused she wastes paper; if an electrician is confused he dies). Possibly also it is because computer science has had to create notations (programming languages) to communicate with computers and has let the same mindset leak into notations to communicate with one another!

No matter the reasons, notation is very important. However, this doesn't mean that there is no ad hoc development of notations. Indeed in some areas of computing, both in programming languages and in formal areas, it seems as if every paper has a new notation. The difference is that the notation itself is seen as significant: the end, not just the tool. The few notations that have become widely used have earned their originators cult status and found their adherents in often hostile factions. Notation is the religion of computer science!

In mathematics notations are most widely used to talk about general properties of classes of mathematical structure. In contrast, computing notations are most often developed to analyse or specify the behaviour of particular systems. Specification, the production of a precise description of an existing or intended system, has become almost synonymous with 'formal methods'.

kinds of notation

The formal notations that have been influential in computing fall into two main camps:

finite process – notations capturing the sequences and processes of computation in terms of a finite number of stages, phases, states or steps. This includes many diagrammatic representations such as state transition networks and flowcharts, and also textual notations such as formal grammars and production rules.

infinite state – more obviously mathematical notations using either set and function notation, or a more 'algebraic' representation. The most common of these is 'Z' which was originally developed at the programming research group at Oxford [[S88]].

One of the enduring problems in computing has been representing and reasoning about *concurrent activity*. One thing at a time is difficult enough, but several things happening at once spell analytic breakdown! However, this is particularly important for HCI as even a simple interface involves the human and computer working together. Collaboration and networking add to this,

In fact, one of the oldest diagrammatic notations in computing is Petri Nets which was designed to represent concurrent and parallel activity [[P62,PNW02]]. This has a small, but continuing and dedicated community. Probably more influential (possibly because they look more like programming languages) have been the more algebraic notations for concurrency, CCS and CSP [[H85,M80]]. These cover very similar ground to one another and, perhaps because of this, have been the source of some of the deepest internecine strife in computing. CCS has been the basis for an ISO standard notation LOTOS [[ISO89]]. Both Petri Nets and LOTOS have been used extensively in HCI (see below).

4. "Detailed" description

In this section we'll look at how different aspects of mathematical, formal and algorithmic methods can be used in HCI.

two plus two – using simple calculation

Straightforward mathematical calculations are everywhere in HCI. These range in complexity. At the simplest are simple arithmetic, for example, in the GOMS keystroke-level model [[CMN80,CMN83]]. The models behind information foraging theory (see chapter 7) are more complex, using differential equations. In areas such as visualisation, information retrieval and graphics, mathematics is again central.

Even small 'back of the envelope' calculations can be surprisingly effective in helping one to understand a problem. I recall some years ago thinking through the statement (which I'm sure you've seen as well) that graphical displays have high 'bandwidth'. This obviously has to be interpreted in terms of visual perception, not just raw pixels per second, but I'll accept it for output. But what about input, do screen buttons and icons increase that? In fact, a quick Fitts' law calculation shows that no matter what the number and size of the screen buttons a reasonable typing speed is always faster (see box). The difference is that whereas the lexicon of the keyboard is fixed and has to be interpreted by the user in any context, GUIs can be contextual, showing appropriate actions (if you know any information theory, this is a form of adaptive compression). Notice that a small mathematical argument can lead to a design perspective.

Back of the Envelope

Compare keyboard with screen for rate of entry measured in bits per second.

Keyboard:

Take typing times from KLM times quoted in [[DFAB98]].

nos targets – 64 keys

good typist – 9 keys per sec.

rate = $9 * \log_2(64)$ = 54 bps

Screen:

Screen width W with items size S on it. The average distance to target is half the width. To make calculations easier assume a square screen and that the screen is completely filled with targets.

Fitts' Law – $0.1 \log_2 (D/S + 0.5)$

$D = W/2$

nos items – $(W/S)^2$

$$\text{rate} = \frac{\log_2 ((W/S)^2)}{0.1 \log_2 (W/2S + 0.5)}$$

$$\approx \frac{2 \log2 (W/S)}{0.1 \log_2 (W/S)}$$

$$= 20 \text{ bps}$$

So screen clicking is nearly three times slower than typing!

Let's work through a similar example. Often the 7+/–2 rule [[M56]], which is about working memory, is mistakenly over applied. One example is for menu systems and you may well have seen suggestions that the number of menu items per screen (e.g. on a web page) shouldn't exceed 7+/–2. On a touch screen large targets are easier to hit, again suggesting that small numbers of larger targets are a good idea. However, the fewer menu items on a single screen the more menu levels that are required to find particular content. Let's assume there are N items in total and you choose to have M menu items per screen. The depth of the menu hierarchy, d, is given by:

$$d = \log N / \log M$$

If we look at a single display, the total time taken will be the time taken to physically display the screen and the time taken for the user to select the item, all times the number of levels:

$$T_{total} = (T_{display} + T_{select}) \times d$$

Using Fitts' law for T_{select} and the formula for d, we get:

$$\begin{aligned} T_{total} &= (T_{display} + A + B \log M) \times \log N / \log M \\ &= ((T_{display} + A)/ \log M + B) \times \log N \end{aligned}$$

Notice that the effect of the increased number of screens exactly balances the gains of larger targets and the only factor that varies with the number of menu items is the per screen costs ($T_{display}$ + A). This suggests that the more items per screen the better. Look at virtually any portal site and you'll see that practical experience has come to the same conclusion!

In fact there are extra factors to consider; for very small targets Fitts' law starts to break down which puts lower limits on target size. Also errors are very significant, as this causes wasted screen displays, so smaller numbers of well explained items may be better. For larger numbers of items a further factor sets in, the time taken for the user to locate an item on the display. There is evidence that for linear menus the actual select time is closer to linear than logarithmic. Redoing the calculations shows that this visual search becomes the limiting factor for larger screens, leading to a maximum menu size depending on refresh time (which is still much larger than 7+/-2 for most cases!) However, good design of screen organisation and sub-headings can push this visual search back towards the logarithmic case (see [[LC98]]). For WAP with small scrolling displays the figures are again different.

Notice that being precise forces us to make assumptions explicit, and also, by focusing us on the critical factors, helps us look for potential design solutions.

detailed specification

One of the main uses of formal methods in HCI, as in other areas of computing, has been to specify the behaviour of specific systems. There are three reasons for doing this:

- to analyse the system to assess potential usability measures or problems
- to describe the system in sufficient detail so that the system that is implemented is what the designer intends
- as a side effect of the above, the process of specification forces the designer to think about the system clearly and to consider issues that would otherwise be missed

Some of the earliest work in this area used formal grammars, in particular Reisner's work on BNF [[R81]]. The strand of work arising from this is described in detail in chapter 6.

Very closely related to this is the widespread use of diagrammatic representations of dialogue structure including the uses of STNs (as in section 2). Diagrammatic formalisms are often seen as being less formal with implications (depending on your viewpoint) of being (i) less rigorous or (ii) more accessible. It is certainly the case that for many people diagrams are more immediately appealing, but they can be just as rigorous as more obviously 'mathematical' looking notations. The main 'informality' of many graphical notations is in the labels for boxes and arcs, which are typically very important for understanding the real meaning of the diagram, but not interpreted within the formalism. As we'll see in the case study in section 5, this is actually a very powerful feature.

Newton's Principia Mathematica

The idea that graphical=informal would have been very strange to the Greek geometers and many (not all) mathematicians for whom diagrams are critical for understanding. When Newton wrote his treatise on gravity and motion, 'Principia Mathematica', he deliberately used geometric explanation rather than more textual notations, because he thought the mathematical explanation would make things too easy. Only really clever people would be able to understand the diagrams!

As noted earlier, human–computer dialogue typically involves multiple, potentially concurrent strands of activity. Even something as simple has a dialogue box may have many button groups, each easy to discuss individually, but all can be used in an arbitrary order by the user. The HCI group at Toulouse pioneered the user of an object-based variant of Petri Nets, ICO (Interactive Cooperative Objects), for specifying user interface properties [[PB95]]. Recall that the Petri Net formalism was designed precisely to be able to manage concurrent activity. Petri Nets have a long-standing analytic theory which can be used to analyse the resulting representations of systems, used both in Toulouse and by others who have adopted Petri Nets. The Toulouse group have worked for some years using their formalism in the design of air-traffic control interfaces and have recently created design and analysis tools for the approach [[NPPSB01]].

Another very successful approach to concurrency has been the use of LOTOS at CNUCE in Pisa. It is interesting to note, however, that this work has found greater external interest since the emphasis shifted from full use of the LOTOS notation to a diagrammatic representation of task hierarchies, called ConcurTaskTrees (CTT), where groups of sub-tasks are linked using concurrency operators from LOTOS (making more precise the plans in standard HTAs [[S95]]). The original intention of this was as a bridge towards full use of LOTOS, but has gradually stolen the limelight. Again a critical feature has been the introduction of a support tool CTTE [[P00,PS00,PMG01]].

These are all notations and formalisms where the principal focus is on the flow of dialogue. Others have used state oriented notations such as Z to specify systems. One of the earliest examples of this is Sufrin's specification of a 'display editor' (in contrast to command line) back in 1982 [[S82]]. Algebraic notations have also been used [[TGGCR96]]. I find that these full specifications really force one to think hard about the system that is being designed.

A simple example I use with students and in tutorials is a four function calculator. What is in the state? Clearly there is a number that is currently displayed, but many students get stuck there. The parts that have no immediate display are harder to think about as a designer. However, this hidden state is also more confusing for the user when it behaves unexpectedly, so it is more important that the designer gets it right.

Calculator scenario

```
user types:   1 + 2  7 = - 3
start after   1 + 2
```

action	**display**	**pend_op**	**total**
	2	+	1
digit(7)	27	+	1
equals	28	none	28
op(−)	28	−	28
digit(3)	**283 !!!**	−	28

Some of the extra detail becomes apparent when you think about particular user actions – when the '=' key is pressed the calculator must know what operation to do (+,-,*,/) and what the previous value was, so the state must record a 'pending' operation and a running total.

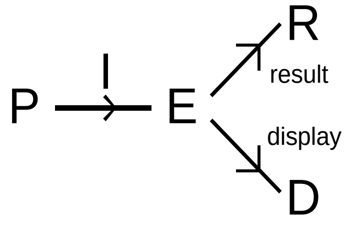
The other method I find very useful is to play through a scenario annotated with the values of various state variables. Doing this to the calculator shows that an additional flag is needed to distinguish the case when the display says '2' because you just typed '2' or because you just typed '1+1'. In the former case typing '3' would give you '23', in the latter it would be '3'.

Of course, formal notations are not very useful unless they give some benefit. In the hands of an expert analyst, the process of producing a detailed specification will reveal new insights on the usability of the system. However, formal specification also allows the opportunity for more automated analysis. Automatic proof tools have been used, but are still difficult to use. Because of this, there has been a growing interest in model checking tools that enable (still quite simple) properties to be checked automatically [[AWM95,PS00]]. One advantage of these is that if a property fails then the normal output of the checking tool is a trace of user actions that would cause the violation – a form of crude scenario that can be fed back to those not expert in formal techniques.

Dialogue specifications were used extensively in the early user interface management systems (UIMS) [[P85]. For a period these were less extensively used, partly because the ready availability of event or object-based GUI builders lead to a more 'try it and see' model of UI construction. However, there has been a growing research agenda in computer-aided design and construction of user interfaces (for example, the CADUI conference series).

A significant area within this has been 'model-based' user interface development environments including UIDE [[FS95]], MASTERMIND [[SSCMS96]] and TADEUS [[ES95]]. These have some sort of model of the user's task, the underlying application semantics and the device capabilities. The designer creates the models using a notation or design environment and then the system automatically generates the actual user interface. This is particularly powerful when targeting multiple devices – for example, PDA, WAP, web as well as standard desktop interfaces.

modelling for generic issues



The area of formal methods with which I am most associated personally, and which perhaps is the beginning of formal methods in HCI as an identifiable area, is the strand of modelling started in York in the mid-1980s, typified by the PIE model [[DR85]].

The PIE model takes a very general view of an interactive application as a transformer of the history of user actions (for historical reasons labelled P) into a current display and possibly also some kind of additional result (printout, saved document). This model and variants of it were used to specify and analyse a range of general user interface issues, mainly to do with the users' ability to predict future behaviour based on the available display and their ability to reach desired goals in the system [[D91]].

This use of formal methods is particularly powerful as one is able to make inferences about properties of a whole class of systems. The cost of working with very precise models is defrayed over many systems which are instances of the general class.

Other examples of this kind of activity include work on understanding case-based systems as used in help desks [[DH95]] and the modelling of authoring environments for educational media [[K97]]. Abstract formal modelling has even been used to formalise some of the cognitive dimensions properties found in chapter 5 [[RS96]].

A topic that has been studied intermittently from the very first PIE paper [[DR85]] onwards has been undo. This is partly because it is a generic issue and so a good one to deal with using such models and partly because the nature of undo makes it a good candidate for formalisation. Some systems try to make undo reverse the effect of any previous command, including itself. Quite early on we were able to show that (with the exception of two state systems like light switches!) this was impossible and that undo has to be regarded as a different kind of 'meta-action'. There has been a substantial amount of work since then, including group undo (see below). However, the most complete treatment of undo has been in Mancini's thesis [[M97]] which captures the multi-level nature of undo. The same model 'the cube' based on a form of the PIE model, was also used to model back and history mechanisms in hypertext and the web, showing how formalism can expose and capitalise on similarities between disparate phenomena [[DML97,DM97]].

CSCW and groupware

Formal modelling and specification work has not been limited to single user interaction. As well as specifications of particular systems there are a number of areas that have had more extensive analysis. One example is workflow packages, which are based upon various kinds of models of collaborative processes. Here the formal representation is largely in order to build a system and this is reminiscent of the use of formal dialogue notations in early UIMS.

An area that has required deeper formal treatment is distributed editing and group undo. The problems with the latter are mainly due to 'race' conditions when two people edit parts of the document virtually simultaneously and messages between their computers pass one another on the network [[EG89, D95]]. Group undo problems are perhaps more fundamental

Group undo

Alison and Brian are working together on a report using a collaborative editing application. Alison pastes a paragraph into the report, realises she has not put it where she wants and is about to select 'undo' when the phone rings. While she is on the phone Brian, in his office, notices the paragraph looks a little strange and so edits it and the surrounding text to make it flow. Alison finishes her phone call and, without looking at her screen, continues where she left off and presses 'undo'. What happens? Does this undo Brian's last action? Does it remove the paragraph she pasted, in which case what happens to the edits that Brian has done to it?

as there are similar implementation issues and it is also very unclear what is the right thing to happen anyway! [[AD92]]

Although there have been a number of papers dealing with specifications of particular CSCW issues (e.g. [[PB96]]), there are few notations aimed specifically at this area. CTT is an exception as it has been extended to explicitly deal with several people cooperating on a task [[P00]]. My own LADA notation is another example [[D95b]].

As well as using formal methods to talk about the groupware systems themselves, there has been some work in using formalisms to understand the human side of collaboration. Winograd and Flores' use of Speech Act theory embodied in the Coordinator tool is one example [[WF86]]. Belief logics and related formalisms [[EW94,D94]] have also been used to formalise the shared and disparate beliefs and knowledge of collaborating people. This could be seen as a formal parallel of the sort of shared understanding described in chapter 10. Another example of this type of work has been the use of artificial life and game theory to understand collaboration [[M94]].

time and continuous interaction

In applied mathematics, for example simple mechanics, one typically deals with continuous phenomena such as the location and velocity of a ball:

$$\frac{dv}{dt} = -g \qquad\qquad \frac{dx}{dt} = v$$

In contrast, computing tends to deal with discrete values at clocked time points. Instead of rates of change we look at state changes at particular events or on clock ticks. Certain types of system include discrete computer controllers interacting with continuous real world phenomena, for example, a fly-by-wire aircraft, or an industrial controller. A branch of formal specification has grown up to deal with such phenomena, called hybrid systems [[G93]], but it is very much a minority area compared with the discrete formal methods community. The hybrid systems area also draw a very sharp distinction between discrete computer components and continuous physical ones.

Even in desktop interfaces there are phenomena that are continuous, such as the movement of the mouse, and others that we perceive as continuous, such as the location of an icon we are dragging. For many years I have argued that these status phenomena are important and should be dealt with on a par with event phenomena [[D91,D91b,DA96,D98]]. More recently, virtual reality, ubiquitous computing and similar areas have further emphasised the importance of these phenomena and various strands of modelling work have ensued including an EU project, TACIT, focused on continuous phenomena, which used notations from the hybrid systems area[[TCT, DMF01]] and other work based on systems theory [[W99]].

A related area that has always been a hot topic personally is time. Delays and timing have long been seen as important in HCI [[S84]]. However, there was a period from roughly the mid-1980s to mid-1990s when it was assumed that personal computers were getting faster and faster and so problems due to time would go away. This led to implicit design assumptions I called the "myth of the infinitely fast machine" [[D87]]. It was an area I have returned to several times in formal and semi-formal material; [[DRW98,**url-ref**]] until, largely due to delays in the web, it became more widely clear that computers wouldn't just get 'fast enough'. Recent formal work in the area has included attempts to deal with multiple granularity in time [[KBN00]].

paradigms and inspiration

There has been a rich interplay between cognitive and computational models since the earliest days of computing. Cognitive models have influenced AI research and neural networks, and also AI models and computer-like architectures have been used to model human cognition. There is an even longer Pygmalion tradition of constructing humanoids in clockwork, steam, flesh enlivened with electricity – looking to the latest technology in the hope of capturing the essence of the animus that makes us human. The human as computer model is perhaps simply the latest in this line, but has been informative and successful despite that.

Computational analogies can similarly be useful as a way of understanding various interface phenomena.

If we assume users have perfect knowledge of the system and its behaviour they should be able to work out an optimal route through the system to their goal. This can be seen as an AI planning model of human interaction and is the implicit model behind a variety of optimal path interaction metrics [[TCJ01]].

However, much of interaction is more contingent. We sort of know what we want and when we see it we may recognise it or at least see that one state of the system is closer than another. Our interaction with the system is more of a goal seeking behaviour, seeking a partly understood goal within a partly understood system. For such systems search and optimisation algorithms may be better models than planning.

Search and optimisation is a huge area within computing, and is complex partly because many of the problems are intractable (in the sense of exponential time) – you cannot consider every possible solution. One class of methods is called hill-climbing. You take a partial solution and look at all closely neighbouring solutions. If all of them are less good then your current solution is (locally) the best. If any are better, you take the best of the solutions you considered as your new current solution. This is like walking in the hills and always taking the steepest upward path. Eventually you come to a hill top. The problem with steepest-ascent hill-climbing is that you are only guaranteed to get to some hill top, not the biggest. If you start in Cambridge (UK) you are likely to end up at the top of the Gog-Magog hills (approximately 100 metres); if you start at Kathmandu, the peak you end up at will be much higher! Locations such as the Gog-Magog hills, which are higher than anything close, but not highest of all, are called *local maxima*.

One solution to this, which has been very powerful in AI game playing algorithms, is to find a *heuristic* [[FD96]]. This tells you not how good a partial solution is, but how good further solutions in a particular direction are likely to be. For example, in chess you can score a board position based on the value of the pieces. With a suitable heuristic one can perform hill-climbing (or variants), but based on the heuristic, not the actual value. In real hill climbing, instead of looking at the heights just one step away, one scans the horizon to see which direction looks more mountainous.

In a user interface this reminds us how important it is for users to be able to predict what will happen if they perform actions. Direct manipulation and graphical interfaces are particularly bad at this because vision is a here-and-now sense (see my own discussion of visual vs. 'nasal' interfaces in [[D96]]). Surprisingly few visualisation systems give you this sort of 'over the horizon' view to enable a heuristic search. Two exceptions are HiBrowse [[EGP94]], which allows you selectively drill-down hierarchical attributes in a database, telling you how many items would be selected if you choose particular values, and Influence Explorer [[TSDS95]], a dynamic slider-based visualisation which has histograms of selected items and also (differently coloured) items that just miss one or two selection criteria. At a broader level information scent (chapter 7) is a form of heuristic.

Another good solution to avoid local maxima in the computer domain is adding randomness, either just starting at random locations and doing a simple hill-climb, or being a bit random in one's exploration, not *always* taking the best path (e.g. simulated annealing). Thimbleby et al. model interfaces as finite graphs and use various graph theoretic properties to analyse potential usability [[TCJ01]]. Their methods include constructing Markov models which measure how likely users are to make particular state transitions (or perform particular actions depending on the current state). In various interfaces they find that random strategies perform better than typical user behaviour, because users have mistaken models and hence planning strategies fail! This is also one of the reasons children perform better with computer systems and household appliances than adults, they are less afraid to experiment randomly. (See my randomness web pages for more about algorithms that exploit stochastic properties [[D02d]]).

Search is just one example where we can draw analogies between algorithms and interfaces to give us inspiration for design (and for algorithms). However, search is special, not just because goal-directed exploration is a very general interface issue, but also because the process of design itself is a form of goal directed search.

Consider the normal pattern of prototyping-oriented design:

- think of an initial design
- evaluate it
- consider alternatives to fix bugs or improve the design
- select the best
- evaluate that
- ... iterate ...

In fact this is precisely hill-climbing behaviour and has exactly the same problems of local maxima – designs which cannot be incrementally improved, but are not necessarily that good.

A really good designer will have some idea of the final solution, based on experience and insight, and so the initial start point is more likely to be a 'Kathmandu' prototype than a 'Cambridge' one.

Given not all designers are so inspired, traditional graphical design practice usually includes several diverse start-points before commencing more incremental improvements – a form of hill-climbing with random start-points.

However, it is analytic and theoretical approaches that can give us the ability to have heuristics, assessing more accurately the potential directions. This is particularly important in novel areas where we have little experience to guide our intuition.

So, if you are a brilliant designer in a well-trodden area – do what you like.

Otherwise, a prototyping-based UI development process is essential as we don't understand users or interfaces well enough to produce the right system first time, but ...

Prototyping is fundamentally flawed unless it is guided by an analytic and theoretical framework.

socio-organisational Church–Turing hypothesis

I'll conclude this section by discussing another area where computation can act as a paradigm or inspiration.

An organisation has many aspects: social, political, financial. However, amongst these diverse roles, many organisations perform an information processing role: transforming orders into delivery instructions and invoices, student exams into degree certificates. Recall that the Church–Turing thesis postulates that all computation is fundamentally equivalent. This only covers what is possible to achieve, not how it happens, which of course is very different!

However, practical experience shows that all practical computational devices tend to exhibit similar structures and phenomena. This is why models of physical computation have proved so effective in cognitive modelling and vice versa. If we apply this to an organisation we get the *socio-organisational Church–Turing hypothesis*, a term coined by Wilkinson, Ramduny-Ellis and myself at a workshop in York some years ago [[DWR98,D02]]:

socio-organisational Church–Turing hypothesis

If we look at the information processing aspects of an organisation, we would expect to see similar structures and phenomena to those in mechanical computation devices

For example, computers have different kinds of memory, long term memory (hard disks) and short term (RAM and processor registers). In an organisation the former is typically explicit in terms of filing cabinets of paper or computer systems, the latter is often held in people's heads or scribbled notes on jotters.

A special part of the memory of a computer system is the program counter, which says what part of a program the computer is currently executing. The organisational analogy is the state of various formal and informal processes. If we look for these *placeholders* for the state of organisational processes, we typically find them either in people's memory or to-do lists, or in the physical placement of papers and other artefacts. That is the office ecology is part of the organisation's working memory; cleaners who tidy up are brainwashing the organisation!

computational parallels

	computer	cognitive model	organisation
process	program	procedural memory	processes
data	data	LTM	files
placeholder	program counter	STM/activation	location of artefacts
initiative	interrupts, event-driven	stimuli	triggers

5. Case Study – dialogue specification for transaction processing

As my case study I'm not going to use a recent example, but one from over 15 years ago, in fact before I became a computing academic, and before I'd even heard the term HCI! At the time I was working for Cumbria County Council working on transaction processing programs in COBOL, the sort of thing used for stock control or point-of-sale terminals in large organisations.

Why such an old example, rather than a more sexy and up-to-date one? Well first because this sort of system is still very common. In addition, the issues in these large centralised transaction processing systems are very similar to those of web based interfaces, especially e-commerce systems. Thirdly, it is a resounding success story, which is not too common in this area, and a 1000% performance improvement is worth shouting about. Finally and most significantly, because it was a success, it gives us a chance to analyse why it was so successful and what this tells us about using formalism today.

The other thing I ought to note is that although this was a very successful application of formal methods in interface design, I didn't understand *why* at the time. It is only comparatively recently that I've come to understand the rich interplay of factors that made it work and so perhaps be able to produce guidelines to help reproduce that success.

background – transaction processing

Transaction processing systems such as IBM CICS have been the workhorses of large-scale information systems, stock management and order processing since the late 1970s. They are designed to be able to accept many thousands of active users.

Architecturally these systems are based around a central server (or cluster) connected to corporate databases and running the transaction-processing engine (see figure 4). In the system I worked with this was an ICL mainframe but in web-based applications it will simply be a web server or enterprise server. The user interacts with a form-based front-end. In the systems I dealt with in the mid-1980s the front-end was semi-intelligent terminals capable of tabbing between fields. Subsequently, in many areas these were replaced by PCs running 'thin client' software and now are often web-based forms. The centralisation of data and transaction processing ensures integrity of the corporate data, but the fact that users interact primarily with local terminals/PCs/browsers means that the central server does not have to manage the full load of the users' interactions.

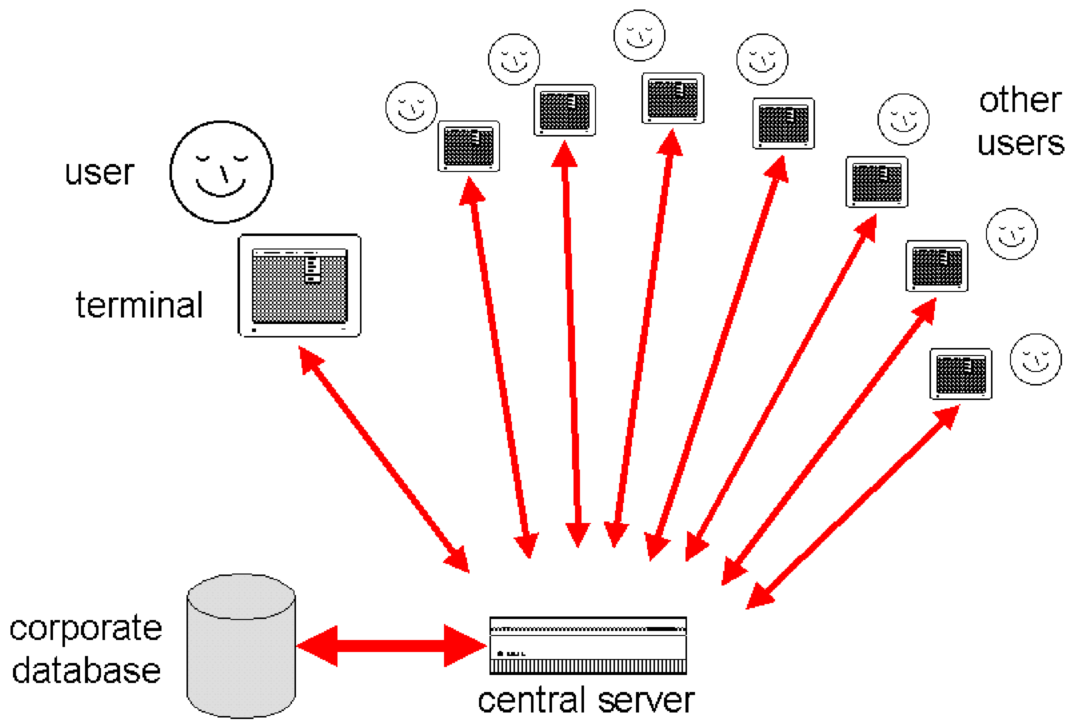


Figure 4. physical architecture of transaction processing system

When the user interacts the succession of events is as follows:

① user fills in form on terminal

② terminal may perform some initial validation (e.g. numbers vs. letters, range checks, date format, or, on thin PC client or Javascript on web forms, more complex validation)

③ user checks and then submits form (presses special key or screen button)

④ terminal/PC/browser sends form data as a message to the transaction processing engine (e.g. CICS or web server) on the central server

⑤ transaction-processing engine selects appropriate application module for message (based on last screen/web page or information in message)

⑥ application module interprets message (form data), does further checks, performs any database updates, gets any required data from the database and generates a new screen/web page as 'result'

⑦ transaction processing engine passes this back to the terminal

⑧ terminal presents the screen/web page to the user

All these stages except ⑥ are managed by the transaction-processing infrastructure. This sounds as if the job in designing this part should be straightforward, most of the complexity of dealing with detailed user interactions has been dealt with. But it is not quite so simple as all that ...

the problem...

In a GUI or any single user interface, the succession of events in the program is straightforward:

- user event 1 arrives (e.g. mouse press)
- deal with event and update display
- user event 2 arrives (e.g. mouse release)
- deal with event and update display
- user event 3 arrives (e.g. key click)
- deal with event and update display

As we know this can cause enough problems!

In a transaction processing system, with one user, the application module may receive messages (with form data) in a similar fashion. However, the whole point of such systems is that they have many users. So, the module may receive messages from different users interleaved:

- message 1 for user A received
- deal with message and generate new screen/web page for user A
- message 1 for user B received
- deal with message and generate new screen/web page for user B
- message 2 for user B received
- deal with message and generate new screen/web page for user B
- message 2 for user A received
- deal with message and generate new screen/web page for user A

The transaction processing engine deals with passing the new screens back to the right user, but the application module has to do the right things with the form data in the messages. In the case of simple transactions, this may not be a problem, for example, if the application simply allows the user to enter an ISBN number and then returns data about the book, the system can simply deal with each message in isolation. However, a more complex dialogue will require some form of state to be preserved between transactions. For example, a request to delete a book may involve an initial screen where the user fills in the ISBN, followed by a confirmation screen showing the details of the book to be deleted. Only then, if confirmed, will the system actually do the deletion and generate a 'book has been deleted' screen. Even a search request that delivers several pages of results needs to keep track of which result page is required and the original search criteria.

Getting back to my workplace in Cumbria in the mid-1980s, the transaction systems in place at that stage only dealt with the simple stateless record display transactions or multi-page search transactions ... and even the latter had problems. When several users tried to search the same database using the system they were likely to get their results mixed up with one another!

I was charged with producing the first update system. Whilst occasionally getting someone else's search results was just annoying, deleting the wrong record would be disastrous.

all about state

So what was wrong with the existing systems and how could I avoid similar, but more serious problems? In essence, it is all about state.

In most computer programs you don't need to worry too much about state. You put data in a variable at one stage and at a later point if you require the data it is still there in the variable. However, in the case of transaction processing modules the module may be re-initialised between each transaction (as is the case with certain types of web CGI script), so values put in a variable during one transaction won't be there at all for the next transaction. Even worse, if the same module is dealing with several users and not re-initialised, then values left behind from a transaction for one user may still be 'lying around' when the next user's transaction is processed. This is precisely what was happening in the search result listings. Some of the state of the transaction (part of the search criteria) was being left in a variable. When the system was tested (with one user!), there was no problem, but when several users used the system their search criteria got mixed up. Although it was possible to explicitly save and restore data associated with a particular terminal/user, the programmers had failed to understand what needed to be stored. Instead, the existing programs coped by putting state information into fields on the form that were then sent back to the next stage of the transaction. With web-based interfaces similar problems occur with session state.

However, there is also a second, more subtle part of the state: the current location in the human–computer dialogue.

In traditional computer algorithmics, the location in the program is implicit. It is only when one starts to deal with event-driven systems, such as GUIs, network applications and transaction processing, that one has to explicitly deal with this. And of course traditional computer science training does little to help. Not only are the principal algorithms and teaching languages sequential, but also the historical development of the subject means that sequential structures such as loops, recursion, etc., are regarded as critical and in lists of essential competency, whereas event-driven issues are typically missing. Even worse, event-based languages such as Visual Basic and other GUI development languages have been regarded as 'dirty' and not worthy of serious attention. Possibly this is changing with Java becoming a major teaching language, but still the event-driven features are low on the computer science agenda!

So, computer programmers in the mid-1980s as well as today were ill prepared both conceptually and in terms of practical skills to deal explicitly with state, especially flow of control.

This was evident in the buggy transaction modules I was dealing with. The flow of the program code of each module looked rather like a twiggy tree, with numerous branches and tests that were effectively trying to work out where in the flow of the user interaction the transaction was situated.

```
if  confirm_field is empty  // can't be confirm screen
                  // or user didn't fill in the Y/N box
then if  record_id is empty  // must be initial entry
     then prepare 'which record to delete' screen
     else if valid record_id
          then read record and prepare confirm screen
          else prepare error screen
else if  confirm_field = "Y'
     then if  record_id is empty  // help malformed
          then prepare error screen
          else  if valid record_id
                 else do deletion
                 then prepare error screen
     else if  confirm_field = "N'
          then prepare 'return to main menu' screen
          else prepare 'must answer Y/N' screen
```

No wonder there were bugs!

Of course, if one looks at many GUIs or web applications the code looks just the same ... Try using the back key or bookmarking an intermediate page in most multi-stage web forms and you'll probably find just how fragile the code is.

the solution

A flow chart of the program looked hideous and was very uninformative because the structure of the program was not related to the structure of the user interaction. So, instead of focusing on the code I focused on the user interaction and produced flowcharts of the human–computer dialogue. Figure 5 shows a typical flowchart.

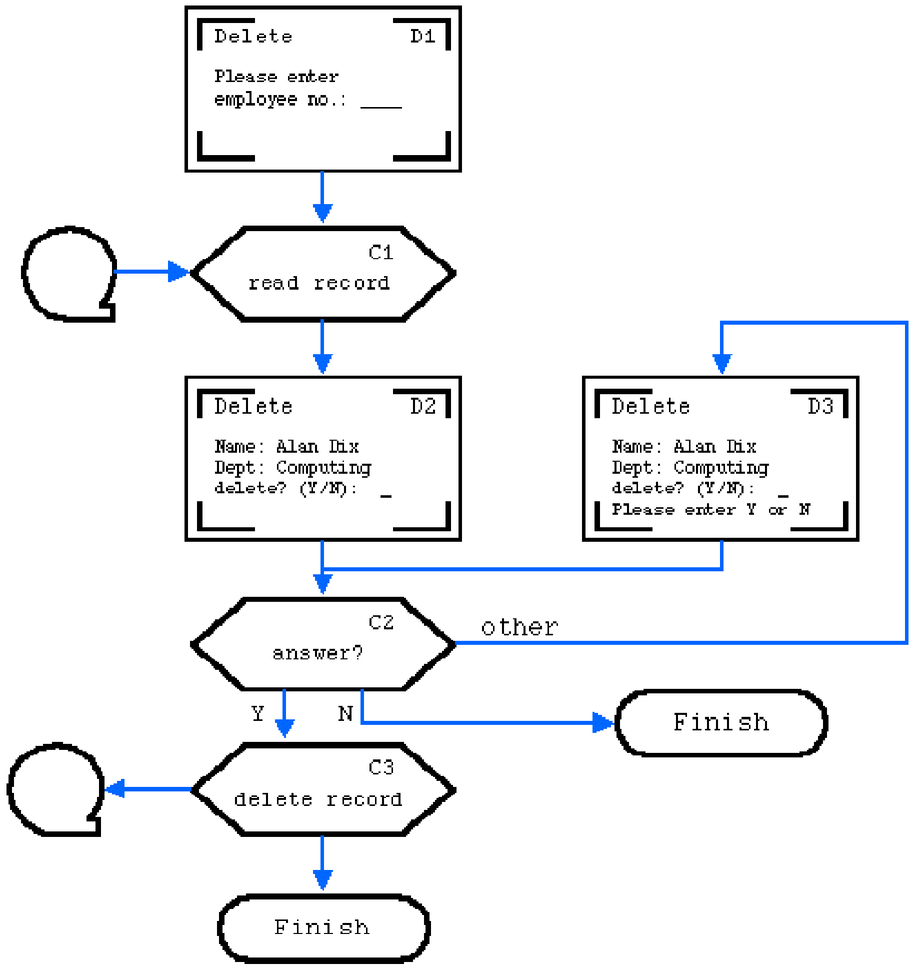


Figure 5. flow chart of user interaction

Each rectangle represents a possible screen and a miniature of the screen is drawn. The angled boxes represent system actions and the 'tape' symbols represent database transactions. Note that this is not a flowchart of the program, but of the human–computer dialogue, rather like the STNs in section 2. Note also that the purpose is to clarify the nature of the dialogue, so the system side is only labelled in quite general terms (e.g. 'read record'). These labels are sufficient to clearly say what should happen and do not say how the code to do this works in details. This is because the difficult thing is getting the overall flow right.

Notice also that each major system block and each screen is labelled: D1, D2, D3 for the screens, C1, C2, C3 for the system code blocks. These are used to link the flowchart to boilerplate code. For each screen there is a corresponding block of code, which generates the screen and, very importantly, stores the label of the next system block against the terminal/user. For example, screen D3 will save the label 'C2'. The first thing the module does when asked to deal with a message is to retrieve the label associated with the user. If this is blank it is the start of the dialogue (generate screen D1 in this case), otherwise the module simply executes the code associated with the relevant system block.

This all seems very mundane, but the important thing is that it worked. Systems that were taking months to develop could be completed in days and the turnaround time for upgrades and maintenance was hours. That is systems were being produced at least 10 times faster than previously and furthermore with less bugs!

why it worked ...

So why is it that such a simple formal method worked so well and can we use this to assess or improve other formalisms or develop new ones?

Let's look at some of the features that made it function well:

useful – addresses a real problem!
The notation focused on the overall user-interface dialogue structure that was causing difficulties in the existing systems. So often formalisms are proposed because they have some nice intrinsic properties, or are good for something else, but do not solve a real need.

appropriate – no more detailed than needed
For example, there was no problem in producing the detailed code to access databases etc., so the formalism deals with this at a very crude level 'read record', 'delete record' etc. Many formalisms force you to fill in lots of detail which makes it hard to see the things you really need it for as well as increasing the cost of using it.

communication – mini-pictures and clear flow easy to talk through with client
Formal methods are often claimed to be a means to improve communication within a design team, because of their precision. However, when precision is achieved at the cost of comprehensibility there is no real communication.

complementary – different paradigm than implementation
It is quite common to use specification methods that reflect closely the final structure of the system. For example, object-oriented specification for object-oriented systems. Here, however, the specification represents the structure of the dialogue which is completely different from the structure of the code. This is deliberate: the notation allows one to see the system from a different perspective, in this case one more suitable for producing and assessing the interface design. The relationship between the structure of the notation and the structure of the code is managed via simple rules, which is what formalisms are good at!

fast pay back – quicker to produce application (at least 1000%)
I have been involved in projects where substantial systems have been fully specified and then implemented and have seen the improvements in terms of quality and *long-term* time savings. However, I still rarely use these methods in practice even though I know they will save time. Why? because I, like most people, like instant pay-back. Spending lots of time up-front for savings later is very laudable, but when it comes to doing things I like to see results. Not only that, but clients are often happier to see a buggy partial something than to be told that, yes, in a few months it will all come together. The dialogue flowcharts didn't just produce long-term savings, but also reduced the lead time to see the first running system.

responsive – rapid turnaround of changes
The feeling of control and comprehension made it easier to safely make changes. In some formal methods, the transformation process between specification and code is so complex that change is very costly (see [[DH89]] for a discussion of this). The assumption underlying this, as in much of software engineering, is that well specified systems will not need to be changed often. Of course, with user interfaces, however well specified, it is only when they are used that we really come to fully understand the requirements.

reliability – clear boilerplate code less error-prone
Although the transformation process from diagram to code was not automated, it was a fairly automatic hand process applying and modifying boilerplate code templates. This heavy reuse of standard code fragments greatly increases the reliability of code.

quality – easy to establish test cycle
The clear labelling of diagrams and code made it easy to be able to track whether all paths had been tested. However, note that these are not just paths through the program (which effectively restarted at each transaction), but each path through the human–computer dialogue.

maintenance – easy to relate bug/enhancement reports to specification and code
The screens presented to the user included the labels, making it easy to track bug reports or requests for changes both in the code and specification.

In short the formalism was used to fulfil a purpose, and was, above all, neither precious nor purist!!

6. Current status

To see where formal methods are going in HCI, we'll start off looking back at the progress of formal methods first in computing in general and then in HCI. We'll then move on to examine the trends and potential ways forward in the area.

retrospective – formal methods in computing

There was a period, in the late 1980s and very early 1990s, when every major research project in computing *had* to have a formal methods component to be respectable. There were known problems in industrial take-up, but this was widely explained as due to lack of suitably trained personnel. As a new breed of formally adept graduates entered the market, formal methods would find its true place as central to computer systems design.

Even by the end of this period the excuses were wearing thin and certainly by the mid-1990s the popular spirit had moved against formalism: "doesn't scale", "too difficult", "requires too much training"... Although there is still a strong formal methods community the emphasis is more on particular domains where safety, cost or complexity make the effort of formal methods worthwhile. Within these areas there have been a continual stream of industrial success stories and some consultancy firms make formal methods expertise one of their market differentiators (see Clarke and Wing's 1996 survey article [[CW96]]).

The critique is all sensible, and 'problems' in the formal methods community were largely due to over-hype in the early days. In other fields, for example structural engineering, one would not expect to apply the same level of analysis to a garden gate as the Thames flood barrier in London. Even within a single structure the analysis of a ship's hull would be different from an internal cabin door, and a porthole beneath the waterline may be different from one above deck level.

However, there is another side to this. In so many disciplines that have been fashionable and then fallen into disrepute, one hears, usually from within the discipline itself, that 'it has never been used in practice' and has failed somehow to meet (overblown) expectations. However, again and again one finds that it is because the discipline constantly redefines itself to exclude those parts that have become successful and 'spun off'. For example, artificial intelligence experienced the same hype and decline, but spin-off areas such as natural language processing, computer vision, and aspects of expert systems are all widely used (and used in HCI!).

Looking with less exclusive eyes, we can see a huge growth in the use of structured methods and diagrammatic specifications, especially in the use of UML. Many of the notations used have their roots in what would have been called 'formal', but now is perhaps simply normal. Note too that it is precisely because systems are large and complex that more formal notations are used. It is easy for programmers to get the details right, but they need support in understanding interactions and architecture. Of course, if formal methods are defined to exclude all this, then, true, adoption is low!

retrospective – formal methods in HCI

Within HCI formal methods have always had a rougher ride as the rigid and cold notations stand in contrast to the richness of human interaction. The area gained somewhat due to the early general formal methods hype, and certainly that was one of the enablers for the early York work. However, rather than growing dramatically and then suffering reverse (as was the case within computing), the formal methods sub-community within HCI has gradually grown and is now able to support not only an annual conference largely of its own (DSVIS), but a number of other conferences where it is a major facet (CADUI, EHCI, TAMODIA).

Although individuals have very different agendas and are wedded to their own methods, the relatively small FM in HCI community has tended to knit together against the 'opposition' from the wider HCI community and so has managed to avoid some of the internal schisms that have bedevilled FM in computing. This has meant that the collections, journal special issues and conferences in the area have included a wide range of formalisms including logics, set theoretic specifications, diagrammatic notations and even cognitive task models.

Formal methods has always been criticised for only dealing with toy problems, and to a large extent this would also be a valid criticism of a large proportion of the FM in HCI literature. A significant number of papers describe new specification techniques, developments of old ones, or applications of existing methods and then apply them to simple case studies. This is perhaps reasonable given the need to start simple in order to address problems and issues one at a time. And, there are signs of maturity, as some of the longer established groups and methods (e.g. Toulouse and Pisa) have created support tools and are applying their techniques to problems in large commercial organisations.

We are perhaps still looking for headline success stories to sell formal methods into the mainstream HCI community in the way that the NYNEX study did for GOMS [[GJA92]], but it seems as if this is now on the horizon.

The culture clash between post-modern HCI and formalism has always been stronger in the US than elsewhere, and formal papers in CHI are very rare. However, there has been quite a strong line of papers in CSCW conferences and HCI journals using formal arguments and analyses to deal with issues of distribution and concurrency. For example, in describing the use of 'operation transforms' and undo/redo frameworks for managing race conditions and conflicts in group editors [[AD92, PK92, PK94, CD95, D95, RNG96, SJZYC98, RG99, S00]]. The various 'spatial' models in CSCW are also reliant to varying degrees on formal definitions [[BBFMR94, R96, SBB97, DRDTFP00]]. These are both cases where the underlying area is complex and simpler arguments and explanations fail.

The last of these is an example of creating generic models/specifications to address a broad issue rather than a specific system. I have always found these the most exciting papers, giving the biggest bang for buck, as the understanding gained can be applied to many different systems. However, these publications are relatively rare. Producing such work requires a comfort in using the formalisms and an ability to look at an area in very broad terms. It may be that these skills are usually found in different personality types, hence making it difficult for an individual to succeed. However, it may equally well be due to education – it is only in the more 'techie' courses that more formal methods are taught (and not always there). The FM in HCI community is therefore full of those who (and I caricature) either started off in very technical areas and are gradually learning to understand people or started off in more people-oriented areas and gradually learning that hand waving is not enough. This of course takes quite a time to mature, but, when it does, means that the FM in HCI community contains the most well-rounded people in IT!

prospective

So where does this take us?

The areas which have greatest take-up are where the formalisms are simplest (as in my case study) and where there is tool support. Particularly powerful are cases where the formalism can be used to generate interfaces, either early prototypes or as part of the final running system (c.f. appropriateness and fast feedback in section 5). This is set to continue; for example CNUCE's CTTE is being used in a growing number of sites.

However, it is also clear that there are application areas which are intrinsically complex, such as the work on distributed group editing and group undo. This area is hitting new complexity limits as the various researchers widen the types of operations they deal with, and it seems that some simplifying framework is needed.

Looking more widely, the trend of having lots of papers about different methods is likely to continue. Each research group has its own focus and there is a good feeling about working on 'your' notation. Rather than fighting human nature it would be good to see the development of an integrating framework to make the relationships between different notations and methods more clear. Perhaps more importantly, such a framework is essential to allow interoperation between the growing numbers of formally based interface design and analysis toolkits.

Note that this is still a wish as there is no real sign of such an integrating framework at present; the closest may well still be the original York modelling work! Perhaps the closest is the work on the syndetic modelling framework [[BMDD00]], which proposes that systems are regarded as lower-level interactors each described by appropriate micro-theories and then bound together into higher-level interactors by macro throries. However, this is more a farmework for formulating a broad theory, not the theory itself.

In fact, prompted by the writing of the early drafts of this chapter I started this process of seeking a unifying framework between formal using traces of activity as a common point between widely different notations [[D02b]].

The existing safety critical application areas, such as flight control, are likely to remain central, with perhaps other high cost-of-failure applications such as e-commerce joining them.

The web, WAP and other aspects of global networking are creating new problem areas that are difficult to understand:

- every system is becoming a distributed system, with portions on a user's machine, running as applets, at multiple interacting servers (consider .Net)
- multiple users, machines and applications engender emergent properties

- state, which we have seen is difficult, is broken up over servers, URL parameters and multiple frames and windows in a user's browser
- parts of the system are unreliable or insecure
- there are generic components (browsers, web servers) as well as specific applications
- the back button and bookmarks mean that applications keep 'restarting' in the middle!

These all suggest that formal models and reasoning may be not optional but essential! Note that the most recent general collection in this area, Palanque and Paternó's 1997 book [[PP97]], used the web browser as a common example – and things have got a lot more complex since then. We are still awaiting the equivalent of an MVC or Seeheim model of interfaces in this type of environment and similar issues to those in group editing and undo may be necessary.

The other area that is fast growing in importance is the very small – multiple context dependent ubiquitous devices dynamically linking and reconfiguring themselves ... hopefully to do something useful! There has been relatively little work on formal issues in this area although some colleagues and I have addressed aspects of context and location modelling [[DRDTFP00]]. Clearly this is another challenge for which, again, more formal analysis will be essential.

Both global networking and ubiquity lead to emergent properties which may mean that areas such as artificial life or the still growing area of critical and complex systems (in the Santa Fe sense) will be needed to understand or model (perhaps in a simulation sense) the situations.

Architectures are also a problem for ubiquitous systems with low-level device events needing to be marshalled and converted into higher-level events for applications. Those working in the area are talking about the need for a Seeheim-style architecture and there has been progress with workshops dedicated to this issue [[RDA02]].

Since my earliest work in formal methods for HCI, issues of time and status phenomena have been critical [[D91,D91b,DA96,D98]]. Whilst the dominant interfaces were event driven this was largely regarded as a marginal concern. However, now we have ubiquitous devices, with physical co-ordinates, biomedical sensors [[A02,AW02]], multi-media and multi-modal inputs ... not so marginal anymore.

Although some of the work is just waiting to be re-applied there are a number of open problems:

- non-localised semantics – many continuous-stream inputs such as voice or a hand gesture have meaning over a period of time, not at an instant, but do not have hard boundaries to those periods
- multiple granularity of time – large granularity time periods (e.g. today) do not map simply onto finer grain times (do we mean working day: 8am-5pm, waking day: 7am-1am, etc.). There are first steps in this area [[KBN00]], but it is a complex issue.
- ideas of location – in the room (does in the doorway count), close vs. far (how close?)

Note in each case we have an apparently discrete higher-level concept (in the room, today, hand gesture for 'open door'), but it has 'fuzzy' edges when mapped onto lower-level features.

Perhaps this sums up the problem and the challenge of formal methods in HCI. Whenever we capture the complexity of the real world in formal structures, whether language, social structures or computer systems, we are creating discrete tokens for continuous and fluid phenomena. In so doing we are bound to have difficulty.

However, it is only in doing these things that we can come to understand, have valid discourse and design.

7. Further reading

There are only a few full texts explicitly on formal methods in HCI.

My own monograph covers the PIE model and many extensions and other models, including those on which status/event analysis is based.

- Dix, A. J. (1991). Formal methods for interactive systems. Academic Press, London. ISBN 0-12-218315-0
  ToC and some chapters: <http://www.hiraeth.com/books/formal/>

See also my co-authored textbook which has a detailed review of formal dialogue notations in chapter 8 and other formal models in chapter 9:

- Dix, A., J. Finlay, G. Abowd and R. Beale (1998). *Human–Computer Interaction* (2nd ed). Prentice Hall.

Harrison and Thimbleby's 1990 collection includes contributions from many of those working in this area at the time.

- Harrison, M.D. and Thimbleby, H.W., editors (1990). Formal Methods in Human Computer Interaction. Cambridge: Cambridge University Press.

Palanque and Paternó edited a more recent collection, confusingly with the same name as the above! This collection is thematic with the contributors using their various techniques to address the web browser as a common example.

- Palanque, P. and Paternó, F., editors (1997). Formal Methods in Human Computer Interaction. London, Springer-Verlag.

Paternó has also produced a monograph looking at the whole design lifecycle and using ConcurTaskTrees as a central representation.

- Paternó, F. (2000). Model-Based Design and Evaluation of Interactive Applications. London, Springer-Verlag.
  ToC and downloads: <http://giove.cnuce.cnr.it/~fabio/mbde.html>

For an up-to-date view of the field, probably the best source is the DSVIS conference series:

- Design, Verification and Specification of Interactive Systems, SpringerWeinNewYork (1995–2002).

My formal methods in HCI web pages include a bibliography, web links and further resources:

http://www.hcibook.com/alan/topics/formal/

Also see the web page for this chapter, which includes links to online copies of referenced papers and further information refered to in the text:

http://www.hcibook.com/alan//papers/theory-formal-2003/

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