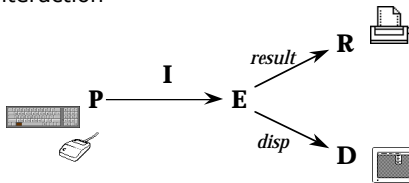


### the PIE model

- 'minimal' model of interactive system
- focused on external observable aspects of interaction



### PIE model - user input

- sequence of commands
- commands include:
  - keyboard, mouse movement, mouse click



- call the set of commands C
- call the sequence P
  - P = seq C

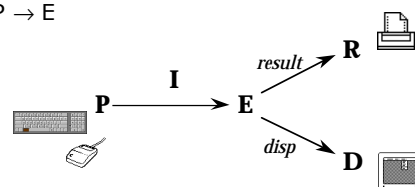
### PIE model - system response

- the 'effect'
- effect composed of:
  - ephemeral display
  - the final result
    - (e..g printout, changed file)
- call the set of effects E

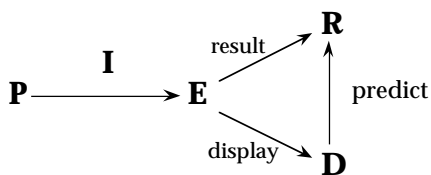


### PIE model - the connection

- given any history of commands (P)
- there is some current effect
- call the mapping the interpretation (I)
  - I: P → E



### properties - WYSIWYG



$\exists \text{ predict} \in (D \rightarrow R) \text{ s.t. } \text{predict} \circ \text{display} = \text{result}$

- but really not quite the full meaning

### state

- avoided to focus external aspects ...
  - ... but often easier to think about!
- but can add it back
  - either 'create' it from I and E
  - or assume E is actually a state

### creating the state!

- given a PIE
- say two command sequences as equivalent if they are indistinguishable in the future:  
 $p \sim q \text{ iff } \forall r \in H: I(pr) = I(qr)$
- the quotient set of P is a minimal state allowing the same effects

N.B. change notation use H (history) instead of P for sequence of commands

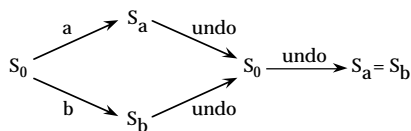
### property of state

- for E to be a state it needs a state update function – call it 'doit':  
 $doit : E \times C \rightarrow E$
- this needs to 'agree' with I:  
 $\forall p \in H, c \in C: doit(I(p),c) = I(pc)$
- if E acts as a state we'll call it S with the initial state  $s_0 = I(<>)$

### proving things – undo

$\forall c : c \text{ undo} \sim \text{null} ?$

only for  $c \neq \text{undo}$



### lesson

- undo is no ordinary command!
- other meta-commands:  
 back/forward in browsers  
 history window

### undo as meta-command

- need to think of ordinary commands C plus augmented commands A:  
 $C^a = C \cup A \quad H^a = \text{seq } C^a$
- also augmented system state:  
 $S^a$
- and behaviour:  
 $doit^a : S^a \times C^a \rightarrow S^a$   
 $I^a : H^a \rightarrow S^a$

### two state (flip) undo

- system keeps two copies of state:  
 $S^a = S \times S$
- ordinary commands update state:  
 $doit^a ( (s_{save}, s), c ) = ( s, doit(s,c) )$
- undo (redo) flips states:  
 $doit^a ( (s_{save}, s), \text{undo} ) = ( s, s_{save} )$

### the real system inside

- the augmented system still needs to be the old system inside!
- link new and old with projection:  
proj:  $S^a \rightarrow S$
- proj(s) is the old state 'inside'

### projected state of flip undo ..

$$S^a = S \times S$$

- projected state simply second component:

$$\text{proj}^a ( (s_{\text{save}}, s) ) = s$$

### properties of flip undo

- undo really reverses undo:  
undo undo  $\sim$  null (strong-uu)
  - undo reverses ordinary commands on projected (original) state  
c undo  $\sim_{\text{proj}}$  null (weak-cu)
- $$\forall s^a \in S^a : \text{proj}(\text{doit}^a (s^a, c \text{ undo})) = \text{proj}(s^a)$$

### stack (multistep) undo/redo

- augmented state is a stack of states:

$$S^m = S^+ \times \text{Nat}$$

$$s^{m_0} = \langle \langle s_0 \rangle, 1 \rangle$$

$$\text{proj}^m : S^m \rightarrow S$$

$$\text{where } \text{proj}^m(\langle \text{hs}, n \rangle) = \text{hs}[n]$$

### stack (multistep) undo/redo

- obvious (!) update:  
doit<sup>m</sup>( $\langle \text{hs}, n \rangle, \text{undo}$ ) =  $\langle \text{hs}, n-1 \rangle$   
if  $n > 1$  - otherwise nothing
- doit<sup>m</sup>( $\langle \text{hs}, n \rangle, \text{redo}$ ) =  $\langle \text{hs}, n+1 \rangle$   
if  $n < \text{length}(\text{hs})$
- doit<sup>m</sup>( $\langle \text{hs}, n \rangle, c$ ) =  
 $\langle \langle \text{hs}[1..n], \text{doit}(\text{hs}[n], c) \rangle, n+1 \rangle$

### properties of multistep undo/redo

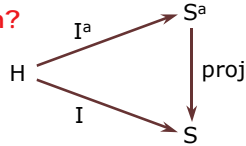
- redo really reverses undo:  
undo redo  $\sim$  null (strong-cu)
- undo reverses commands on projected (original) state  
c undo  $\sim_{\text{proj}}$  null (weak-cu)
- the only way to satisfy these ...

**Prove It !**

### the real system inside (2)

- behaviour?
- if no A commands ever used, identical:  
 $\forall h \in H : \text{proj}(I^a(h)) = I(h)$

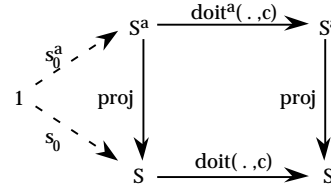
enough?



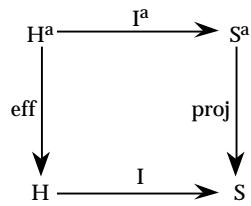
### conservativeness of state

$$\text{proj}(s_0^a) = s_0$$

$$\forall c \in C, s \in S^a : \text{proj}(\text{doit}^a(s, c)) = \text{doit}(\text{proj}(s), c)$$



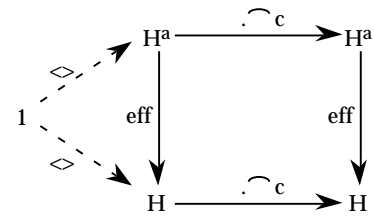
### encapsulation



### conservativeness of effective history

$$\text{eff}(\diamond) = \diamond$$

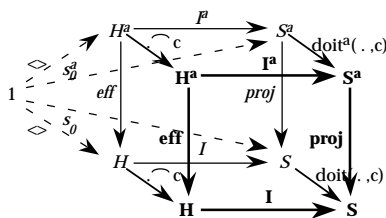
$$\forall c \in C, h \in H^a : \text{eff}(h \wedge c) = \text{eff}(h) \wedge c$$



### the cube

$$\text{eff}(\diamond) = \diamond$$

$$\forall c \in C, h \in H^a : \text{eff}(h \wedge c) = \text{eff}(h) \wedge c$$



### full details ...

R. Mancini (1997).  
**Modelling Interactive Computing by exploiting the Undo.**  
 Dottorato di Ricerca in Informatica, IX-97-5,  
 Università degli Studi di Roma "La Sapienza"